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TWO-TEMPERATURE GREEN AND NAGHDI MODEL ON THERMOELASTC INTERACTION IN AN INFINITE FIBRE-REINFORCED ANISOTROPIC PLATE CONTAINING A CIRCULAR HOLE

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Abstract. The present work is aimed at the study of thermoelastic interactions in an infinite fibre-reinforced anisotropic plate containing a circular hole in the context of a theory of generalized thermoelasticity in which the theory of heat conduction in deformable bodies depends on two different temperatures – conductive temperature and dynamic temperature. The two-temperature generalized thermoelastic model and one-temperature generalized thermoelastic model are formulated on the basis of Green and Naghdi theory of type II (2TGNII). The circular hole surface is assumed to be stress free and is subjected to a thermal shock. The problem is solved numerically using a finite element method. The effects of the presence and absence of reinforcement on the conductive temperature, the dynamical temperature, the stresses and the displacement distributions are studied. A comparison is made with the results predicted by the two theories. Results carried out in this paper can be used to design various fibre-reinforced anisotropic elements under thermal load to meet special engineering requirements.

Key words: Two-temperature theory, Green and Naghdi theory, fibre-reinforced, finite element method.

1. Introduction

Materials such as resins reinforced by strong aligned fibres exhibit highly anisotropic elastic behaviour in a sense that their elastic modulus for extension in the fibre direction are frequently of the order of 50 or more times greater than their elastic modulus in transverse extension or in shear. The mechanical behaviour of many fibre-reinforced composite materials is adequately modelled by the theory of linear elasticity for transversely isotropic materials, with the preferred direction coinciding with the fibre direction. In such composites, the fibres are usually arranged in parallel straight lines. However, other configurations are used. An example is that of circumferential reinforcement, for which the fibres are arranged in concentric circles, giving strength and stiffness in the tangential (or hoop) direction. During the second half of twentieth century, non-isothermal problems of the theory of elasticity become increasingly important. This is due to their many applications in widely diverse fields. First, the high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses that reduce the strength of the aircraft structure. Second, in the nuclear field, the extremely high temperature and temperature gradients originating inside nuclear reactors influence their design and operations (Nowinski [1]). The theory of strongly anisotropic materials has been extensively discussed in the literature, Belfield *et al* [2] studied the stress in elastic plates reinforced by fibres lying in concentric circles. Sengupta and Nath [3] discussed the problem of surface waves in fibre-reinforced anisotropic elastic media. Singh [4] showed that, for wave propagation in fibre-reinforced anisotropic media, this decoupling cannot be achieved by the introduction of the displacement potentials. Hashin and Rosen [5] gave the elastic modulus for fibre-reinforced materials. In classical dynamical coupled theory of thermoelasticity, the thermal and mechanical waves propagate with an infinite velocity, which is not physically admissible. The classical theory of thermoelasticity as exposed for example in Carlson's article [6] has found generalizations and modifications into various thermoelastic models that run under label hyperbolic thermoelasticity, see the survey of Chandrasekharaiah [7] and Hitnarski and Ignazack [8]. The notation hyperbolic reflects the fact that thermal waves are modelled, avoiding the physical paradox of infinite propagation speed of the classical model. In the decade of the 1990's, Green and Naghdi [9]-[11] proposed three new thermoelastic theories based on an entropy equality rather than the usual entropy inequality. Chen and Gurtin [12] have formulated a theory of heat conduction in deformable bodies, which depends upon two distinct temperatures, the conductive temperature and the thermodynamic temperature. For time independent or -dependent situations, the difference between these two temperatures is proportional to the heat supply, and in the absence of any heat supply, the two temperatures are identical. The two temperatures and the strain are found to have representations in the form of a travelling wave plus a response, which occurs instantaneously throughout the body and Warren and Chen [13] investigated the wave propagation in the two-temperature theory of thermoelasticity. Recently, Youssef [14] has constructed a new model of generalized thermoelasticity depending on the two temperatures, where the difference between the two temperatures is proportional to the heat supply with a non-negative constant. Banik and Kanoria [15] studied two-temperature generalized thermoelastic interactions in an infinite body with a spherical cavity. Abbas and Abd-Alla [16] studied the generalized thermoelastic interaction in an infinite fibre-reinforced anisotropic plate containing a circular hole with one relaxation time. Verma [17] discussed the problem of magnetoelastic shear waves in self-reinforced bodies. Chattopadhyay and Choudhury [18] investigated the propagation, reflection and transmission of magnetoelastic shear waves in a self–reinforced media. Chattopadhyay and Choudhury [19] studied the propagation of magnetoelastic shear waves in an infinite self-reinforced plate. Chattopadhyay and Michel [20] studied a model for spherical SH-wave propagation in self-reinforced linearly elastic media. Tian *et al* [21], Abbas [22], Abbas and Othman [23], and Abbas [24] applied the finite element method in different generalized thermoelastic problems.

The exact solution of the governing equations of the generalized thermoelasticity theory for a coupled and nonlinear/linear system exists only for very special and simple initial and boundary problems. To calculate the solution of general problems, a numerical solution technique is used. For this reason, the finite element method is chosen. The method of weighted residuals offers the formulation of the finite element equations and yields the best approximate solutions to linear and nonlinear boundary and partial differential equations (see Wriggers [25]).

This paper considers thermoelastic problem involving such circumferentially reinforced plates. The composite material is then locally transversely isotropic, with the direction of the axis of transverse isotropy now not constant, but everywhere directed along the tangents to circles in which the fibres lie. The problem has been solved numerically using a finite element method (FEM). The conductive temperature, the dynamical temperature, the stresses and the displacement distributions are shown graphically with some comparisons.

2. Problem formulation: Governing equations

In the context of the Green and Naghdi theory with two-temperature and without energy dissipation (2TGNII), the field equations for linear equations governing thermoelastic interactions in a fibre-reinforced linearly thermoelastic anisotropic medium whose preferred direction is that of a unit vector

$\boldsymbol{a}$, in the absence of body forces and heat sources, are as follows [14, 16]:

$$\tau_{ij,j} = \rho \ddot{u}_i, \quad i,j = 1,2,3, \tag{1}$$

$$K^* \varphi_{,ii} = \rho c_e \ddot{T} + T_o \beta_{ij} \ddot{u}_{i,i}, \quad i,j = 1,2,3, \tag{2}$$

$$\tau_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta_{ij} + a_i a_j e_{kk}) + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki})$$

$$+\beta a_k a_m e_{km} a_i a_j - \beta_{ij}(T - T_0)\delta_{ij}, \quad i,j,k,m = 1,2,3, \tag{3}$$

$$\varphi - T = a\varphi_{,ii}, \quad a \geq 0, \quad i = 1,2,3, \tag{4}$$

where ρ is the mass density; u_i the displacement vector components; e_{ij} the strain tensor; τ_{ij} the stress tensor; φ the conductive temperature; $a(\geq 0)$ the two-temperature parameter; T the thermodynamic temperature change of a material particle; T_o the reference uniform temperature of the body; β_{ij} the thermal elastic coupling tensor; c_e the specific heat at constant strain; K^* the material characteristic of the theory; λ, μ_T are elastic parameters; α, $beta$, $(\mu_L - \mu_T)$ are reinforced anisotropic elastic parameters and the component of the vector a are (a_1, a_2, a_3) where $a_1^2 + a_2^2 + a_3^2 = 1$. The comma notation is used for spatial derivatives and superimposed dot represents time differentiation. For circumferential reinforcement, it is normal to employ a system of cylindrical polar coordinates (r, θ, z) and henceforth all components are referred to these coordinates. In this system, for cylindrical symmetric interactions, the displacement vector possesses only the radial component $u = u(r,t)$, where r is the radial distance measured from the origin (point of symmetry), and the stress tensor is determined by the radial stress τ_{rr} and the circumferential stress (hoop stress) $\tau_{\theta\theta}$. For circumferential reinforcement the vector $\boldsymbol{a}$ is everywhere directed in the tangential (i.e. θ) direction, so that in cylindrical polar coordinates $\boldsymbol{a}$ has components $(0,1,0)$. In this case, equations (1)–(4) yield the following governing equations for u, φ and T:

$$\frac{\partial \tau_{rr}}{\partial r} + \frac{1}{r}(\tau_{rr} - \tau_{\theta\theta}) = \rho \frac{\partial^2 u}{\partial t^2}, \tag{5}$$

$$\mathrm{K}^* \left(\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r} \right) = \frac{\partial^2}{\partial t^2} \left(\rho c_e T + T_o \beta_{11} \frac{\partial u}{\partial r} + T_o \beta_{22} \frac{u}{r} \right), \tag{6}$$

$$\varphi - T = a\left(\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r}\right) \tag{7}$$

$$\tau_{rr} = (\lambda + 2\mu_T)\frac{\partial u}{\partial r} + (\lambda + \alpha)\frac{u}{r} - \beta_{11}(T - T_o), \tag{8}$$

$$\tau_{\theta\theta} = (\lambda + \alpha)\frac{\partial u}{\partial r} + (\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta)\frac{u}{r} - \beta_{22}(T - T_o), \tag{9}$$

with$\beta_{11} = 2(\lambda + \mu_T)\alpha_{11} + (\lambda + \alpha)\alpha_{22}$, $\beta_{22} = 2(\lambda + \alpha)\alpha_{11} + (\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta)\alpha_{22}$, where α_{11}, α_{22} are coefficients of linear thermal expansion. For convenience, the following non-dimensional variables are used:

$$\begin{aligned}&(r', u') = \frac{1}{b}(r, u), \quad t' = \frac{c_1}{b}t, \quad (\tau'_{rr}, \tau'_{\theta\theta}) = \frac{1}{D}(\tau_{rr}, \tau_{\theta\theta}), \quad T' = \frac{T - T_o}{T_o},\\ &\varphi' = \frac{\varphi - T_o}{T_o}, \quad c_1 = \sqrt{\frac{D}{\rho}}, \quad D = \lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta.\end{aligned} \tag{10}$$

In terms of the non-dimensional quantities defined in Eqs. (10), the above governing equations reduce to (dropping the dashed for convenience):

$$\begin{aligned}&\frac{\partial}{\partial r}\left(B_1\frac{\partial u}{\partial r} + B_2\frac{u}{r} - B_3 T\right) + (B_1 - B_2)\frac{1}{r}\frac{\partial u}{\partial r}\\ &\qquad + (B_2 - 1)\frac{u}{r^2} - (B_3 - B_4)\frac{T}{r} = \frac{\partial^2 u}{\partial t^2},\end{aligned} \tag{11}$$

$$\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} = \frac{\partial^2}{\partial t^2}\left(\varepsilon_1 T + \varepsilon_2\frac{\partial u}{\partial r} + \varepsilon_3\frac{u}{r}\right), \tag{12}$$

$$\varphi - T = \Omega\left(\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r}\right) \tag{13}$$

$$\tau_{rr} = B_1\frac{\partial u}{\partial r} + B_2\frac{u}{r} - B_3 T, \tag{14}$$

$$\tau_{\theta\theta} = B_2\frac{\partial u}{\partial r} + \frac{u}{r} - B_4 T, \tag{15}$$

where $(B_1, B_2, B_3, B_4) = \dfrac{1}{D}(\lambda + 2\mu_T, \lambda + \alpha, T_o\beta_{11}, T_o\beta_{22})$, $(\varepsilon_1, \varepsilon_2, \varepsilon_3, \Omega) = \left(\dfrac{\rho c_e c_1^2}{k^*}, \dfrac{c_1^2\beta_{11}}{k^*}, \dfrac{c_1^2\beta_{22}}{k^*}, \dfrac{a}{b^2}\right)$.

The surface of the hole i.e. $r = b$ is assumed to be stress free and is subjected to a uniform step in temperature effect so that the boundary conditions are taken as:

$$\tau_{rr}(b,\, t) = 0, \quad \varphi(b,\, t) = \varphi_1 H(t), \tag{16}$$

where $H(t)$ denotes the Heaviside unit step function.

Initially, the medium is at rest and undisturbed and the initial conditions are:

$$u(r,0) = \frac{\partial u(r,0)}{\partial t} = 0,\;\; T(r,0) = \frac{\partial T(r,0)}{\partial t} = 0,\;\; \varphi(r,\,0) = \frac{\partial \varphi(r,\,0)}{\partial t} = 0, \tag{17}$$

3. Finite element method

The Finite Element Method (FEM) is a powerful technique originally developed for numerical solution of complex problems in structural mechanics, and it remains the method of choice for complex systems. A further benefit of this method is that it allows physical effects to be visualized and quantified regardless of experimental limitations. In this section, the governing equations of generalized thermoelasticity based upon two-temperature Green and Naghdi of type II (2TGNII) are summarized, using the corresponding finite element equations. The finite element equations of a generalized thermoelasticity problem can be readily obtained by following standard procedure. In the finite element method, the displacement component $u$and temperature $\varphi$are related to the corresponding nodal values by:

$$u = \sum_{i=1}^{m} N_i u_i(t), \quad \varphi = \sum_{i=1}^{m} N_i \varphi_i(t), \tag{18}$$

where m denotes the number of nodes per element, and N the shape functions. In the framework of standard Galerkin procedure, the weighting functions and the shape functions coincide. Thus,

$$\delta u = \sum_{i=1}^{m} N_i \delta u_i, \quad \delta\varphi = \sum_{i=1}^{m} N_i \delta\varphi_i, \tag{19}$$

With equations (16) and (17), $u' = u_{,i}$ and $\varphi' = \varphi_{,i}$ can be expressed as

$$u' = \sum_{i=1}^{m} N_i' u_i(t), \quad \varphi' = \sum_{i=1}^{m} N_i' \varphi_i(t), \tag{20}$$

$$\delta u' = \sum_{i=1}^{m} N_i' \delta u_i, \quad \delta\varphi' = \sum_{i=1}^{m} N_i' \delta\varphi_i. \tag{21}$$

Thus, the finite element equations corresponding to equations (11)–(15) can be obtained as:

$$\sum_{e=1}^{me}\left(\begin{bmatrix} M_{11}^e & 0 \\ M_{21}^e & M_{22}^e \end{bmatrix}\begin{Bmatrix} \ddot{u}^e \\ \ddot{T}^e \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{Bmatrix} \dot{u}^e \\ \dot{T}^e \end{Bmatrix} + \begin{bmatrix} K_{11}^e & K_{12}^e \\ 0 & K_{22}^e \end{bmatrix}\begin{Bmatrix} u^e \\ T^e \end{Bmatrix} = \begin{Bmatrix} F_1^e \\ F_2^e \end{Bmatrix}\right), \tag{22}$$

where me is the total number of elements. The coefficients in equation (22) are given below.

$$M_{11}^e = \int [N]^T[N]\,dr, \quad M_{21}^e = \int [N]^T\left(\varepsilon_2[N'] + \frac{\varepsilon_3}{r}[N]\right)dr,$$

$$M_{22}^e = \int \varepsilon_1[N]^T\left([N] - \Omega\left\{[N''] + \frac{[N']}{r}\right\}\right)dr,$$

$$K_{11}^e = \int \left[[N']^T\left(B_1[N'] + \frac{B_2}{r}[N]\right) + [N]^T\left(\frac{B_2 - B_1}{r}[N'] + \frac{1 - B_2}{r^2}[N]\right)\right]dr,$$

$$K_{12}^e = \int \left[-B_3[N']^T\left([N] - \Omega\left\{[N''] + \frac{[N']}{r}\right\}\right) + \frac{B_3 - B_4}{r}[N]^T\left([N] - \Omega\left\{[N''] + \frac{[N']}{r}\right\}\right)\right]dr,$$

$$K_{22}^e = \int \left[[N']^T[N] - \frac{1}{r}[N]^T[N]\right]dr, \quad F_1^e = [N]^T\bar{\tau}\big|_1^r, \quad F_2^e = [N]^T\bar{q}\big|_1^r.$$

Symbolically, the discretized equations of equations (22) can be written as

$$M\ddot{d} + C\dot{d} + Kd = F^{ext}, \tag{23}$$

where M, C, K and F^{ext} represent the mass, damping, stiffness matrices and external force vectors, respectively; $d = [u\,\varphi]^t$; $\bar{\tau}$ represent the component of the traction, and $\bar{q}$ represents heat flux. On the other hand, the time derivatives of the unknown variables have to be determined by Newmark time integration method or other methods (see Wriggers [25]).

4. Numerical example

To study the effect of reinforcement on wave propagation, we use the following physical constants for generalized fibre-reinforced thermoelastic materials [15].
$\rho = 2660$ kg/m^3, $\lambda = 5.65\times10^{10}$ N/m^2, $\mu_T = 2.46\times10^{10}$ N/m^2, $\mu_L = 5.66\times10^{10}$ N/m^2, $\alpha = -1.28\times10^{10}$ N/m^2, $\beta = 220.90\times10^{10}$ N/m^2, $\alpha_{11} = 0.017\times10^{-4}$ deg^{-1}, $\alpha_{22} = 0.015\times10^{-4}$ deg^{-1}, $c_e = 0.787\times10^3$ J kg^{-1}deg^{-1}, $T_o = 293$ K.

Before going to the analysis, the grid independence test has been conducted and the results are presented in Table 1. The grid size has been refined and consequently the values of different parameters as observed from Table 1 get stabilized. Further, refinement of mesh size over 10000 elements does not change the values considerably, which is therefore accepted as the grid size for computing purpose.

Table 1. Grid independent test ($\Omega = 0.3$, $t = 0.3$, $r = 2$)

Mesh Size	$\varphi\times10^{-1}$	$u\times10^{-6}$	$\tau_{rr}\times10^{-5}$
500	1.820124	1.273774	−7.397617
1000	1.814828	1.284988	−7.396577
5000	1.812380	1.285377	−7.396564
7000	1.812205	1.285385	−7.396564
10000	1.812204	1.285389	−7.396564

Here, all the variables/parameters are taken in non-dimensional forms. The results for conductive temperature, dynamical temperature, displacement, radial stress and hoop stress has been carried out by taking $\varphi_1 = 1$ and $t = 0.3$.

The first group (Figs. 1–5) shows the differences between the theory of one temperature Green and Naghdi of type II ($\Omega = 0.0$) and the theory of two-temperature Green and Naghdi of type II ($\Omega = 0.1, 0.2, 0.3$). The second group (Figs. 6–10) represent the variations of the physical quantities under two-temperature Green and Naghdi theory of type II (2TGNII) with reinforcement (WRE) and without reinforcement (NRE, when $\alpha = 0$, $\beta = 0$, $\mu_L = \mu_T$). The last group (Figs. 11–15) demonstrate the behaviour of the displacement,

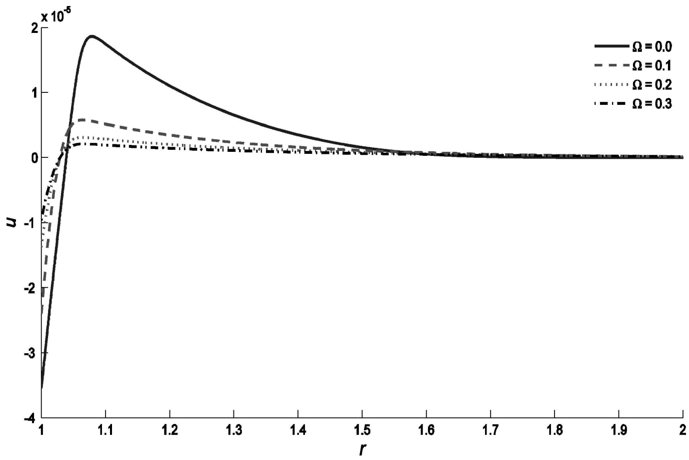

Fig. 1. Variation of u against r for different values of Ω at $t = 0.3$

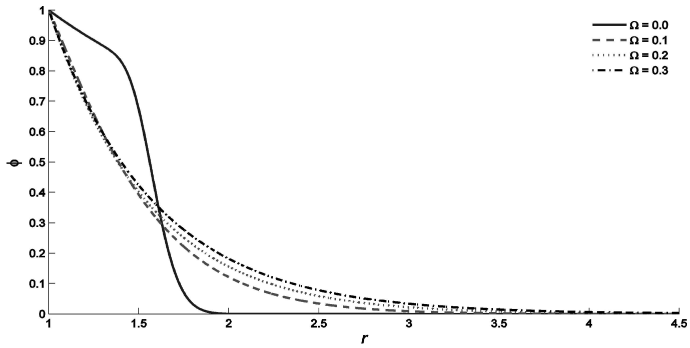

Fig. 2. Variation of φ against r for different values of Ω at $t = 0.3$

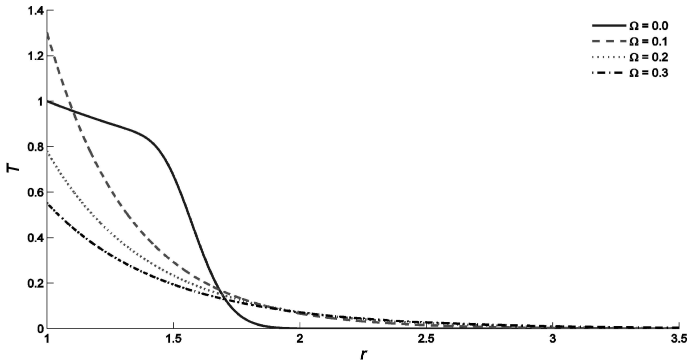

Fig. 3. Variation of T against r for different values of Ω at $t = 0.3$

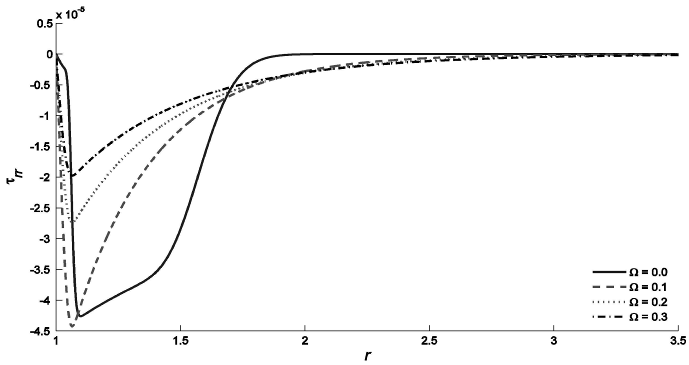

Fig. 4. Variation of τ_{rr} against r for different values of Ω at $t = 0.3$

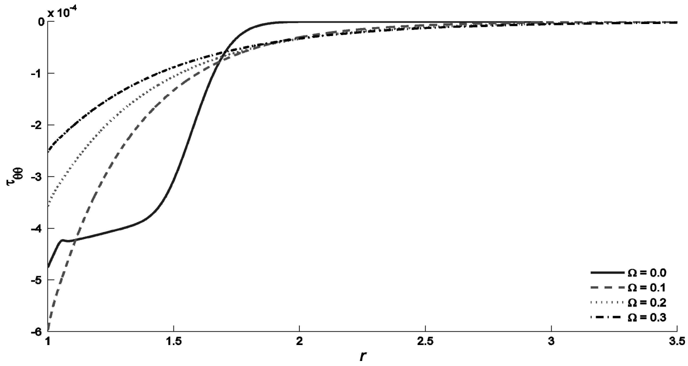

Fig. 5. Variation of $\tau_{\theta\theta}$ against r for different values of Ω at $t = 0.3$

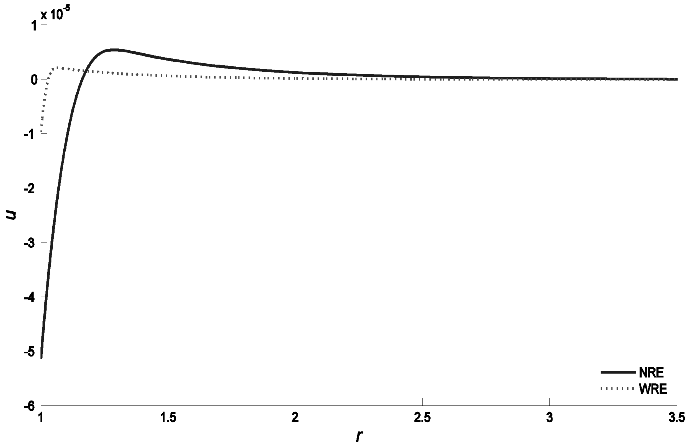

Fig. 6. Variation of u against r at $t = 0.3$ and $\Omega = 0.3$ with (WRE) and without (NRE) reinforcement

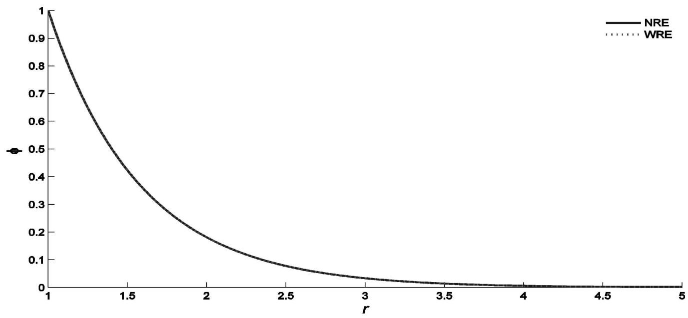

Fig. 7. Variation of φ against r at $t = 0.3$ and $\Omega = 0.3$ with (WRE) and without (NRE) reinforcement

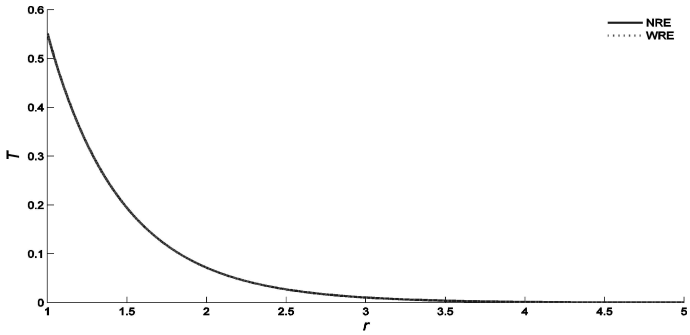

Fig. 8. Variation of T against r at $t = 0.3$ and $\Omega = 0.3$ with (WRE) and without (NRE) reinforcement

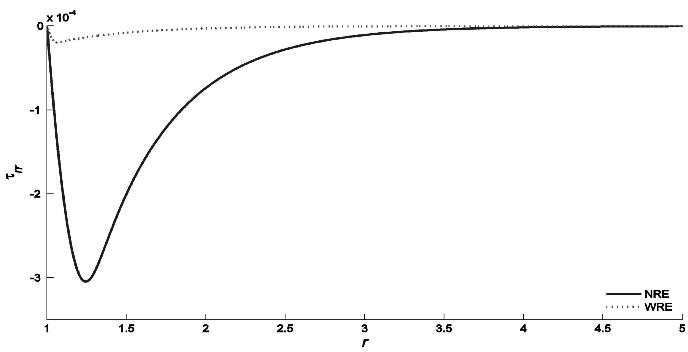

Fig. 9. Variation of τ_{rr} against r at $t = 0.3$ and $\Omega = 0.3$ with (WRE) and without (NRE) reinforcement

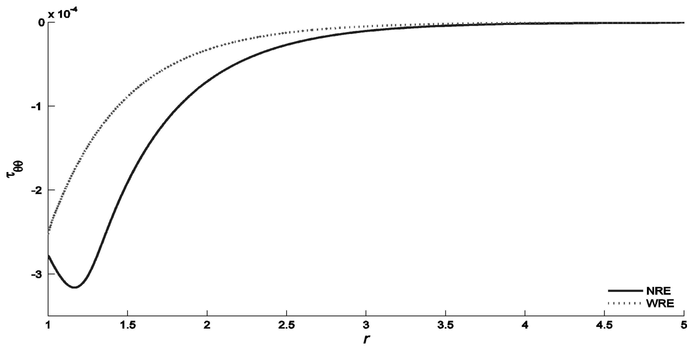

Fig. 10. Variation of $\tau_{\theta\theta}$ against r at $t = 0.3$ and $\Omega = 0.3$ with (WRE) and without (NRE) reinforcement

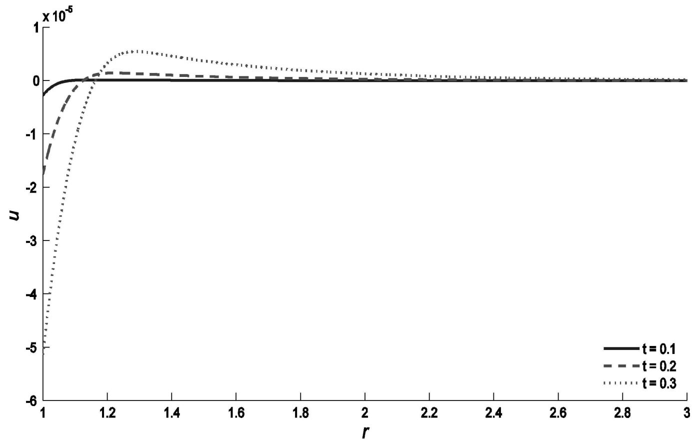

Fig. 11. Variation of u against r for different values of t at $\Omega = 0.3$

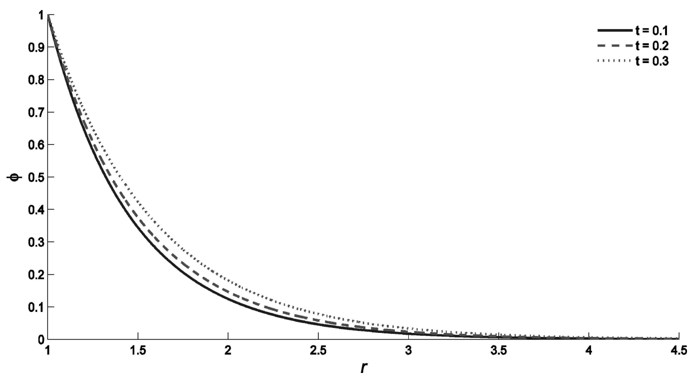

Fig. 12. Variation of φ against r for different values of t at $\Omega = 0.3$

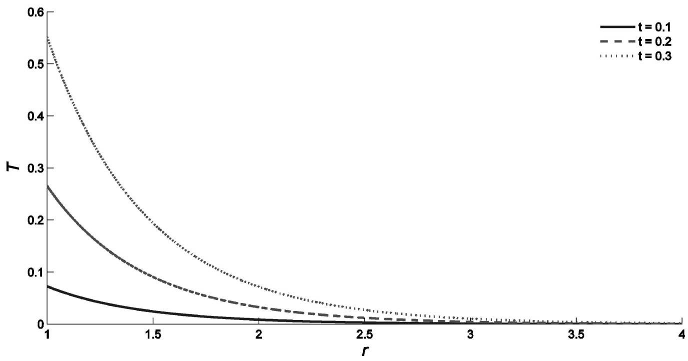

Fig. 13. Variation of T against r for different values of t at $\Omega = 0.3$

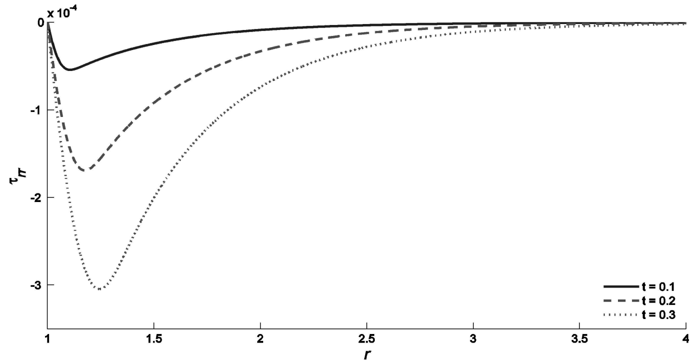

Fig. 14. Variation of τ_{rr} against r for different values of t at $\Omega = 0.3$

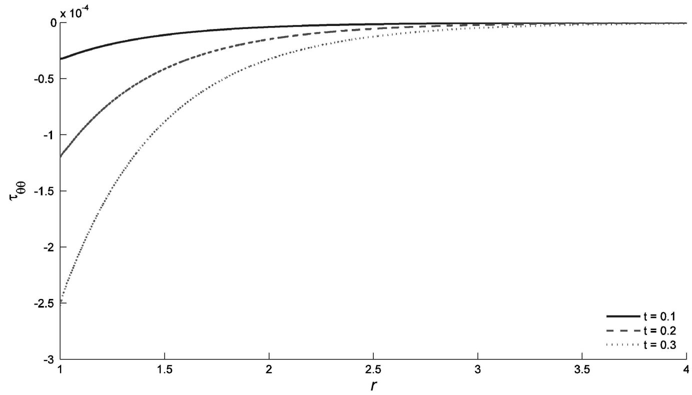

Fig. 15. Variation of $\tau_{\theta\theta}$ against r for different values of t at $\Omega = 0.3$

conductive temperature, dynamical temperature, radial stress and hoop stress under two-temperature Green and Naghdi theory of type II with reinforcement (WRE) for three different values of time.

Figures 1, 6, and 11 show the variation of the radial displacement $u$with radial distance r. This illustrates that the displacement starts with negative values at $r = b$(which is equal to 1), then increases rapidly to positive values. The maximum values of the displacement depend on the values of the time. Figures 2, 7, and 12 show the conductive temperature variation φ with radial distance r. From these figures, one can see that the temperature has a maximum value when $r = 1$ which is satisfied by the boundary conditions. Also, it decreases rapidly with increasing radial distance and vanishes before $r = 4$. Figures 3, 8, and 13 show the variation of dynamical temperature $T$with radial distance r. This result shows that the two type temperature model agree with the generalized thermoelasticity. Figures 4, 9, and 14 show the variation of the radial stress τ_{rr} with redial distance r. At $r = 1$, the stress reduces to zero which agree with the boundary condition. Figures 5, 10, and 15 show that the behaviour of the hoop stress $\tau_{\theta\theta}$ with radial distance r. It is noted that the hoop stress increases to remaining close to zero.

5. Conclusion

The displacement, the conductive temperature, the dynamical temperature and the stresses in an infinite fibre-reinforced anisotropic plate containing a circular hole due to thermal shock have been examined within the framework of the two-temperature generalized thermoelasticity theory under generalized thermoelasticity (Green and Naghdi theory of type II) by using finite element method. The Comparison with predictions was also made in which there was a two-temperature parameter term, and we have found that this parameter has a significant effect on all the fields and on the speed of the wave propagation. So, according to the results of this work, it is important to distinguish between the dynamical temperature and the conductive temperature. The way from the result there is no significant difference in the value of conductive temperature and dynamical temperature are noticed with reinforcement (WRE) and without reinforcement (NRE). The reinforcement has a great effect on the distribution of displacement and stresses.

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